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ABSTRACT

This paper is an attempt to summarize as explicitly as possible certain empirical findings of classical biosystematics and modern semantic ethnography which may be considered to represent formal universals of human mental structure. The paper offers a formal treatment of the subject of taxonomy, and an application of the formalism to several problem areas in the fields of semantics, ethnography, and cognition. The structure formalized here, essentially following the notions of Linnaean biology, is a hierarchy of inclusion relations among a collection of named sets of objects. Section one introduces the formal definition of taxonomic structure and sketches the major outlines of this kind of mathematical object. Section two introduces some theoretical problems relating to taxonomy in ethnographic and semantic contexts and show how this forumlation applies to, and clarifies, these problems. In particular, the notion of semantic contrast in the context of taxonomy is examined in some detail. Section three introduces the notion of taxonomy and examines the nature of the mapping which governs the realization of conceptual taxa by lexical items. (Author/AM)

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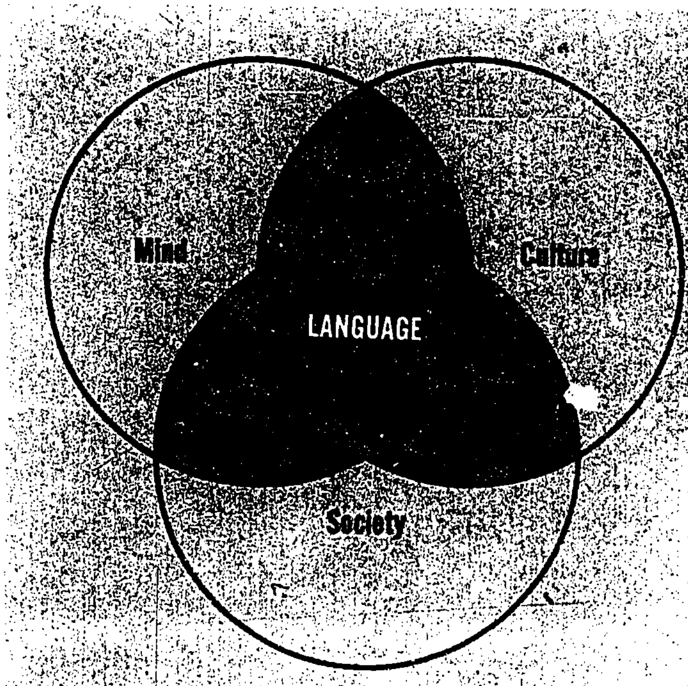

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ON TAXONOMY AND SEMANTIC CONTRAST*

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ON TAXONOMY AND SEMANTIC CONTRAST

Introduction*

The concept of taxonomy is becoming one of increasing importance in the fields of ethnography and semantics. This paper offers a formal treatment of the subject and an application of the formalism to several problem areas in the fields of semantics, ethnography, and cognition.

The expression 'taxonomy' has been used by anthropologists in a variety of ways, some of which bear little relation to the subject of this essay. Two such usages should be mentioned briefly in order that possible confusions may be minimized.

First some anthropologists have used 'taxonomy' to refer to any system of classification and naming regardless of its structure. Used in this way, 'taxonomy' is effectively synonymous with 'lexical domain'. Given the long tradition in biology of applying 'taxonomy' only to a particular structural type of classification system, such a use of the term is unfortunate. 'Taxonomy' and 'taxonomic structure' are defined here so as to be applicable only to a particular subclass of lexical domains that displays certain formal properties.

Secondly, 'taxonomy' is sometimes employed by anthropologists to refer, not to a hierarchy of sets, but rather to an arrangement of properties (semantic features), presumably one felt to be consonant with a hierarchy of sets (Lounsbury, 1964). For example, in the standard usage followed here, if one were to consider the English words person, man, and woman to be involved in taxonomic relations

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*The ideas expressed in this paper were originally stimulated by a letter written by Brent Berlin and Dennis E. Breedlove several years ago. My thoughts on the subject have since developed in constant interaction with Berlin's, and it is neither possible to delimit his contribution to the present essay for separate citation nor to exagerate its magnitude. Detailed comments by H. C. Conklin have also been of value. I would like to thank J. P. Boyd, Ernest Adams and William Geoghegan for useful comments on earlier drafts. P. Raven also read an earlier version of the manuscript. Responsibility for errors is, of course, my own.

the elements of the taxonomic structure would be the set of humans, the set of men and the set of women. On the other hand, in the view suggested by Lounsbury, the basic elements of the taxonomic structure are the properties (features) human, male, female, and so on. It would perhaps be possible to construct a formal account of taxonomic structure based on the property or feature approach. I think, however, that there are strong empirical reasons that militate against the usefulness of such a formal construction, although this is not the place to present those arguments in detail (see Kay, 1966). In any case, the basic elements of taxonomic structures as they are defined below are sets, not properties.

The intuitive notions of taxonomy and taxonomic structure to be formalized here are essentially those of Linnaean biology. The fundamental structure is in brief a hierarchy of inclusion relations among a collection of named sets of objects. The same phenomena have been given formal treatment by Gregg (1954), although in a different way.* Anthropologists such as Frake (1961, 1962), Conklin (1962 a, b, 1964) and Berlin (Berlin, 1969 a, b; Berlin, Breedlove, and Raven 1966, 1968) have found the Linnaean type of structure present in the cultures of non-literate peoples, and it appears likely that important parts of the lexicon of all natural languages are organized taxonomically. The initial discoveries in the modern era that extensive and precise taxonomies exist among illiterate primitives originally occasioned surprise bordering on incredulity in some anthropological quarters. But it is becoming increasingly recognized that the similarity to Linnaen taxonomy of the folk taxonomies discovered by ethnographers and ethnobiologists need occasion no surprise, since Linneaen taxonomy is simply the particular folk taxonomy with which Western Europeans are most familiar. Linnaeus did not invent the principles of taxonomy

*Gregg's more recent formulation (1967) is discussed in the Addendum.

to be employed but merely made explicit those current in his own culture, which, as it turns out, for the most part represent universal principles of classification and nomenclature that are found in all human cultures and languages (See Berlin 1969 a, b with regard to universals in taxonomic nomenclature). The present essay is thus an attempt to summarize as explicitly as possible certain empirical findings of classical biosystematics and modern semantic ethnography which may now be considered to represent formal universals of human mental structure.

Section 1 introduces the formal definition of taxonomic structure and sketches the major outlines of this kind of mathematical object. The mathematical object called a taxonomic structure in the Appendix is quite similar to Gregg's (1954) notion of a taxonomy. The chief differences are (a) the present formulation is simpler (b) the present formulation contains nothing corresponding to Gregg's categories (c) Gregg's formulation contains nothing corresponding to the types of contrast relations defined here. The presentation in Section 1 is informal. References to the Appendix, which contains an axiomatic treatment of the subject, are included in square brackets.

Section 2 introduces some theoretical problems relating to taxonomy in ethnographic and semantic contexts and shows how this formulation applies to, and perhaps clarifies somewhat, these problems. In particular the notion of semantic contrast in the context of taxonomy is examined in some detail.

Section 3 introduces the notion of taxonomy and examines the nature of the mapping which governs the realization of conceptual taxa by lexical items.

Section 1

The purely formal entity which under certain empirical conditions, underlies a taxonomy is called a taxonomic structure. This section is concerned primarily with taxonomic structures. Detailed discussion of what must be added to a taxonomic structure to make a taxonomy is deferred until Section 2. Briefly, the distinction is this: a taxonomic structure is concerned with sets (or classes, or segregates) and the relations among them; it is not concerned with the names these sets may or may not have. In keeping with standard usage, we call the sets (or classes, or segregates) involved in a taxonomic structure taxa (singular taxon). A taxonomy, on the other hand, always includes a taxonomic structure---a set of taxa with certain relations specified among them---and also includes a set of names and a mapping of the set of taxa onto the set of names. We return to this subject in greater detail in the following section.

A taxonomic structure is a relational structure that has two components and that satisfies two axioms. The first component is a finite set T of taxa. Each taxon, is itself a non-null set, that is, a set which has some members. Hence T is a set whose members are non-null sets. Examples of taxa are, the set of all plants, the set of all trees, the set of all oaks (but not the English words plant(s), tree(s), oak(s). In what follows I will use "oak" synonymously with " the set of all oaks" and "oak" synonymously with " the word oak".) Let us call the number of taxa involved in a given taxonomic structure n; we may then enumerate the set of taxa: $T = \{t_1, t_2, \ldots t_n\}$. The set of taxa $T = \{t_1, t_2, \ldots, t_n\}$ is the first component of a taxonomic structure.

The second component is a relation, in particular the relation strict-inclusion-of-sets restricted to the members of T. A set t_i strictly includes another set t_j just if every member of t_j is a member of t_i and there is at least one member of t_i which is not a member of t_j. The set of plants strictly includes the set of trees and also the set of oaks; the set of trees strictly includes the set of oaks. However, the set of Quercus does not strictly include the set of oaks, although it includes it, since the two sets have the same membership, that is, every set includes itself, but no set properly includes itself.

The phrase "restricted to the members of T" indicates that we are only concerned with strict inclusion relations among the sets under consideration, not among all imaginable sets. If the members of T are, say, all the plant taxa, then the relation strict-inclusion-restricted-to-the-members-of-T does not hold between, for example, animal and vertebrate, though it does hold between tree and oak. The second component of a taxonomic structure is thus the relation strict-inclusion-of-sets-restricted-to-the-members-of-T [see Appendix (1)].

Given the relation of strict inclusion restricted to a set T, we define another relation, which is called immediate precedence. Immediate precedence is defined in order to make it easy to express naturally the two axioms for taxonomic structures. Let t_i and t_j, each of which is a set, be distinct members of T. We say t_i immediately precedes t_j just if (i) t_i strictly includes t_j and (ii) there is no other set t_k in T such that t_i strictly includes t_k and t_k strictly includes t_j [Appendix (4)]. For example, tree immediately precedes oak because tree strictly includes oak and there is no other plant taxon which is strictly included in tree and which also strictly includes oak. Speaking loosely, one taxon immediately precedes another when the first is "just above" the second in a taxonomic structure.

We symbolize the two components of a taxonomic structure T (the set of taxa) and $\supset$ (the relation of strict-inclusion-restricted-to-the-members-of-T). Let us represent the ordered pair formed from these two components with the Greek letter tau, $\mathcal{T}$. That is, using angles to enclose an ordered pair, $\mathcal{T} = \langle T, \supset \rangle$ [Appendix (1)].

A relational structure such as $\mathcal{T}$ is a taxonomic structure just if it satisfies the following two axioms [Appendix (1)].

First Axiom: There is exactly one member of T which strictly includes every other member. This member is called the unique beginner. In a taxonomic structure of plants, plant is the unique beginner; it strictly includes each other taxon, such as tree, oak, grass, bamboo, and so on [Appendix (2)].

The second axiom involves the notion 'partition'. A partition is a division of a set into subsets that places each member of the original set in exactly one of the subsets. The subsets are called cells of the partition. For example, suppose the prisoners in a jail are each assigned to a cell so that every prisoner is in some cell and no prisoner is in more than one cell(though different cells may contain varying numbers of prisoners). Then the cells of the jail are the cells of a partition of the set of prisoners.

Let $c(t_i)$ stand for the set of all those taxa immediately preceded by the taxon t_i. For example, if t_i is oak, then the members of $c(t_i)$ are live oak, jack oak, red oak, and so on. Second Axiom: For any taxon t_i in T if $c(t_i)$ has any members, then it is a partition of t_i. This axiom ensures, for a taxon such as oak that has subclasses, that (i) each individual oak in the world is in some subclass and (ii) that each individual oak is in just one subclass. The second axiom does, of course, allow for the possibility that a taxon has no subclasses in T. An example is live-oak (for the author at least) [Appendix (3)].

From this simple axiomatic definition a number of consequences follow that are proved as theorems in the Appendix. These seem to agree rather well with standard notions regarding the formal properties of the structures which underlie taxonomies. Also several definitions can be made which appear to correspond to existing intuitive concepts. Let us begin with some of the latter.

(i) The set of all taxa immediately preceded by the same taxon constitutes a contrast set. That is, a contrast set is any non-null set $c(t_i)$ where t_i is a member of T. In the previous example, all the immediate subclasses of oak; live oak, jack oak, and so on, constitute a contrast set [Appendix (8)].

(ii) A terminal taxon is one that strictly includes no other taxon; that is, a taxon t_i for which $c(t_i)$ is the empty set. For the author, live oak is a terminal taxon [Appendix (11)].

(iii) The level of a taxon is defined as follows: The level of the unique beginner is 0. The level of a taxon immediately preceded by the unique beginner is 1. The level of a taxon immediately preceded by a taxon of level 1 is 2, and so on.* For example, in a taxonomic structure in which the unique beginner, plant, immediately precedes tree, tree immediately precedes oak, and oak immediately precedes live oak; plant is at level 0, tree at level 1, oak at level 2, and live oak at level 3 [Appendix (13)].

(iv) The depth of a taxonomic structure is the greatest (deepest) level attained by any taxon in that structure. In the running example, if no taxon has a level greater (deeper) than live-oak, which is 3, then 3 is the depth of the taxonomic structure [Appendix (18)].

Some of the direct consequences of the above axioms and definitions are the following:

(v) Every contrast set is a proper subset of T and contains at least two members [Appendix (9, 10)].

(vi) Each taxon other than the unique beginner has exactly one immediate predecessor [Appendix (12)].

The following result is the fundamental theorem, as it were, of the theory of taxonomic structure.

(vii) If two distinct taxa have any members in common, then one of them strictly includes the other. That is, any two distinct taxa are either mutually exclusive or in the relation of strict-inclusion. The possible relations between two distinct taxa t_i and t_j in a taxonomic structure are pictured in the Venn diagrams of Figure 1a; the disallowed relation is shown in Figure 1b.

*In general the level of a taxon immediately preceded by a taxon of level n is n + 1.

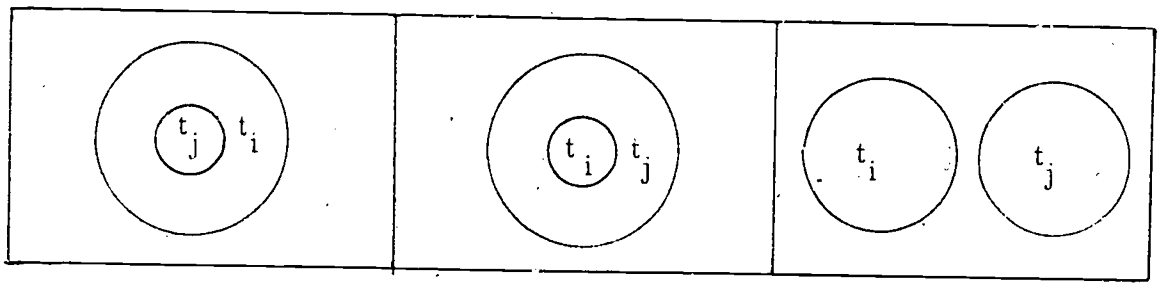


Figure 1a

Possible relations between two distinct taxa

(t_i, t_j)

in a taxonomic structure.

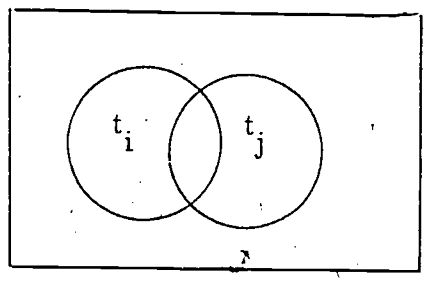


Figure 1b

Disallowed relation between two distinct taxa in a taxonomic structure.

Note that this fact (vii) was not taken as axiomatic but rather is a consequence of the axioms [Appendix (16)]. It corresponds to the intuitive notion that there is no partial overlap between taxa; either one taxon is totally included in the other or they have no member in common. This result (vii) [Appendix (16)] also expresses formally the intuition that taxonomic structures are strictly "hierarchic" and contain no element of "cross-classification." This is the major formal basis for representing taxonomic structures in the standard tree or box diagrams (see Figure 2).

(viii) If a taxonomic structure is of depth n, then it contains at least one taxon at each level from zero to n inclusive. For example, if the depth is five, then there is at least one taxon at each of the levels 0, 1, 2, 3, 4, and 5. In this sense, a taxonomic structure has no gaps [Appendix (19)].

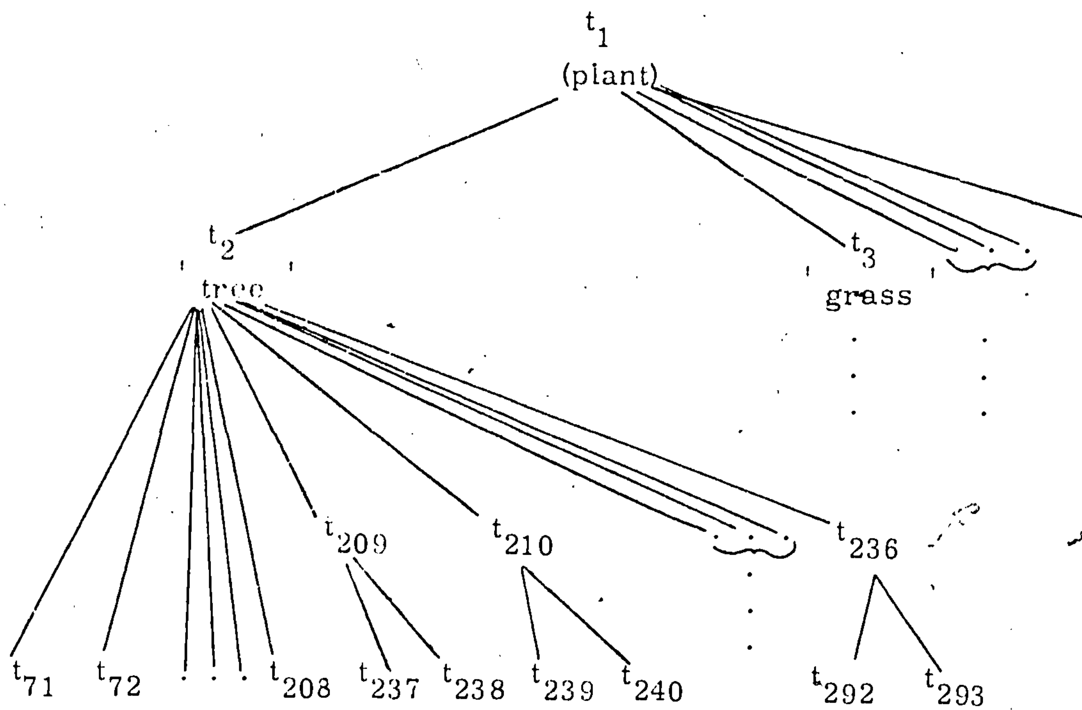
(ix) The terminal taxa constitute a partition of the unique beginner. For example in a taxonomic structure of plants, each individual plant (i.e. each member of the set plant) belongs to exactly one terminal taxon (e.g. live oak, pitch pine, etc.) [Appendix (20)].

(x) Each taxon other than the unique beginner belongs to exactly one contrast set. For example, live oak belongs to the contrast set c(oak), pitch pine to the contrast set c(pine) [Appendix (28)].

The various concepts introduced here are illustrated in Figure 2.

Figure 2a

Taxonomic Structure underlying Tenejapa Tzeltal Plant Taxonomy (1 informant).[(a)]



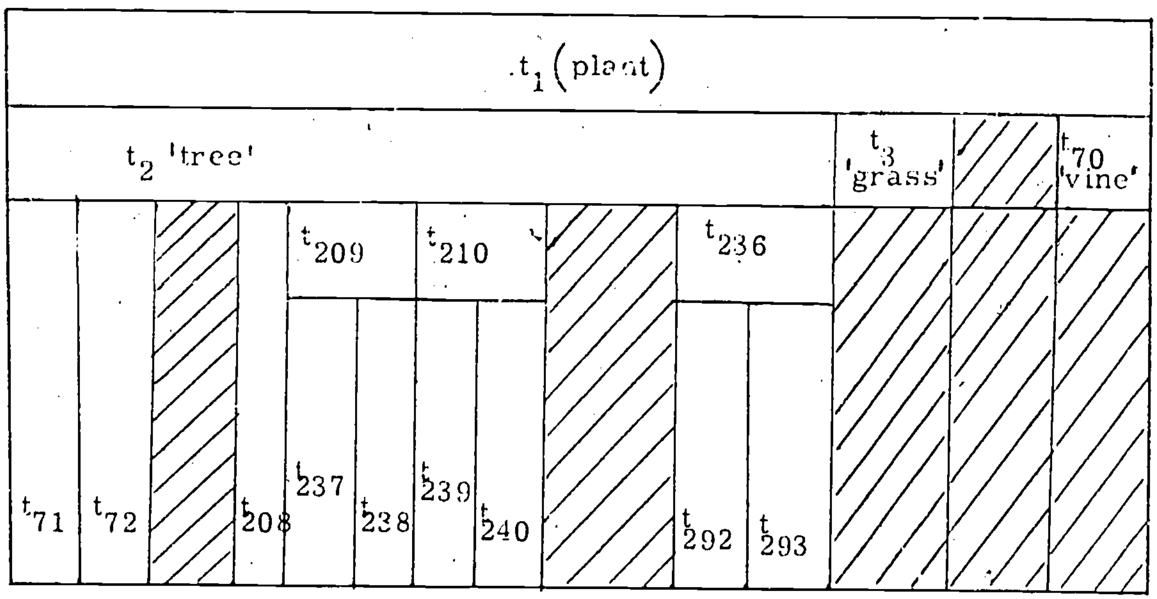
(a) Adapted from Berlin, Breedlove, and Raven (1968). Single quotes indicate glosses of native lexemes. A sequence of three dots indicates omitted detail. The unique beginner, plant, is not lexically realized in Tzeltal. Berlin (1969 a, b) notes that this circumstance is not at all exceptional but rather is characteristic of all but the most advanced developmental stages of natural biotaxonomies.

The following observations, relating to the preceding numbered statements (i - x), may be made concerning Figure 2a.

1. The unique beginner t_1 is not in any contrast set and is the only taxon at level zero.
2. For any taxon t_i :
   (i) For $2 \leq i \leq 70$, t_i is in the contrast set $c(t_1)$ and is at level 1.
   (ii) For $71 \leq i \leq 236$, t_i is in the contrast set $c(t_2)$.
   (iii) For $237 \leq i \leq 238$, t_i is in the contrast set $c(t_{209})$.
   (iv) For $239 \leq i \leq 240$, t_i is in the contrast set $c(t_{210})$.
   (v) For $292 \leq i \leq 293$, t_i is in the contrast set $c(t_{236})$.
   (vi) For $71 \leq i \leq 208$ or $237 \leq i \leq 293$, t_i is a terminal taxon.
   (vii) For $71 \leq i \leq 236$, t_i is at level two.
   (viii) For $237 \leq i \leq 293$, t_i is at level three. (This statement, although true, is not directly inferable from Figure 2a. Taxa $t_{241}, \ldots, t_{291}$ are not pictured. They are in fact all at level three, as each is immediately preceded by a taxon from the set of unpictured, level-two taxa $\{t_2, \ldots, t_{235}\}$.
3. Not all terminal taxa are at the same level.
4. Two distinct taxa at the same level may or may not be in the same contrast set.
5. The depth of the taxonomic structure is not indicated in the Figure. In fact the greatest depth in Tzeltal plant taxonomic structure is obtained within 'beans'.

Figure 2b illustrates the same structure in the type of box diagram often used by anthropologists.

Figure 2b. Box Diagram Corresponding to Figure 2a.



Note: Hatched areas indicate omitted detail. Comparison with Figure 2a reveals that box diagrams are somewhat less precise than tree diagrams in indicating omitted detail.

Section 2

The notion "taxonomy" has been of use to ethnographers and cognitive theorists principally as a means of organizing relations of meaning among items in natural languages and cognitive systems. The organization of meaning relations necessarily entails the notion of (semantic) contrast, and 'contrast' has figured prominently in much recent work in folk taxonomy.

'Contrast' is obviously a relational term, but it has not always been clear in the literature what sort of entities are involved in relations of contrast and what the various types of (semantic) contrast relations are. In particular, despite Conklin's admonition to the contrary (1962: 121), the distinction between classification and nomenclature is sometimes ignored, and consequently it is not clear whether 'contrast' is a relationobtaining between taxa (sememes), or names of taxa (lexemes). Also the term contrast has been combined with 'level' in the phrase 'level of contrast', whose precise signification is far from clear(Cf. Kay, 1966: 21). We return to these issues in greater detail below. Before doing so, however, it is necessary to specify what, in general, we mean by semantic contrast.

Probably we should take the same initial attitude to 'contrast' in semantics as is taken in linguistics generally, namely that one does not assume items to be in contrast with one another unless one finds positive evidence for it. The usual form this evidence takes is that of a frame in which (1) the informant allows substitution of either item and (2) the informant judges the utterances resulting from the alternative substitutions to be different utterances. If the utterance is restricted in length to a single word, this is the method of minimal pairs. Such tests are usually held to be too strong, in the sense that passing them is a sufficient but not a necessary condition for saying that two items contrast. (The problem of deciding which of the informant's responses to take as indicating "this is a different utterance than that" is an important but not an insuperable one.) For semantics we might adopt the following criterion of same/different as a test comparable to the test of minimal pairs, that is one that provides a sufficient, but not a necessary, operational diagnostic for semantic contrast.

(I) Two lexical items can be said to contrast semantically if

(i) there exists an assertion frame in which the informant allows substitution of either item, in the sense that with either alternative substitution he can easily judge the resulting assertion as true or false; and

(ii) the informant's truth judgments for the assertions resulting from the two substitutions are different; that is, one true and one false.

Although we wish theoretically to define relations of contrast between units of content--in this case taxa--our operational criterion must deal with the overt expressions of these units, the lexemes through which the taxa are realized in actual speech. Criterion (I) applies practically to taxa then, to the extent that we can establish empirically the particular taxon which is realized by each test lexeme (lexical item). I return to this problem below.

Speaking somewhat loosely, we may say that (I) boils down to this: two categories contrast semantically if an assertion using one elicits assent from the informant while the otherwise identical assertion employing the other elicits dissent from the same informant. As we noted, this is probably too strong a test. For example the lexical items Morning-Star and Evening-Star might never be shown to contrast on criterion (I) for many informants; yet we might still wish to think of them as contrasting semantically. It suffices to establish here that if two items pass a test like (I), then we have to say that they contrast semantically.

Note that in (I) we have spoken about assertions, not sentences or even declarative sentences. Consider:

(II) (i) That's not a mailman, Johnny, it's a postman.

(ii) I am not an eye-doctor, Sir, I am an opthamologist.

The underlined words in each case may appear to be in semantic contrast in the sense of criterion (I). If, however, one considers the assertions being made rather than the sentences that are their realizations, one sees that these assertions do not concern mail carriers and medical practitioners but rather express preferences about the words mailman, postman, eye-doctor, and opthamologist. These are meta-linguistic assertions in which the words mailman, eye-doctor, etc., occur not in use but in reference. In fact, most native speakers of English will probably accept as a paraphrase of (II, i),

(III) Johnny, 'mailman' and 'postman' mean the same thing, but I prefer that you use 'postman'.

If one accepts the kind of test for contrast given in (I), then clearly statements such as, "the categories plant and tree do not contrast, since all trees are plants", are using 'contrast' in a peculiar sense. In order to retain continuity between semantics and general linguistics, we prefer to keep 'contrast' sufficiently general that at least any pair of items that pass test (I) will be siad to contrast. Evidently, tree and plant will pass many versions of test (I), for example, all those of the form

(IV) All X's have Y

where X is a variable whose values are the taxa tree or plant and Y is any characteristic (or set of characteristics) that distinguish trees from all other plants.

In short, all pairs of items in a taxonomic structure contrast, since assertion frames can always be created for use in test (I) that focus precisely upon the characteristics that distinguish one taxon from the other. Any two distinct taxa must, of course, have different characteristics, otherwise they would have the same membership.

Let us return to the practical problems engendered by a theoretical position that defines contrast as relation between taxa rather than a relation between lexemes. Test (I), narrowly interpreted, cannot be applied to taxa directly, but only to the lexemes that are their names. Nevertheless, the relations of contrast obtaining between taxa may be empirically determined, not only in cases where taxa are lexically realized as polysemous lexemes, but even in those cases which might be supposed *a priori* to present insuperably obstacles, namely those in which there exist several closely related taxa that are not lexically distinguished from each other at all but which nevertheless participate mutually in several different kinds of contrast relations. Nevertheless Berlin and his co-workers have recently demonstrated that such covert categories may be unambiguously identified by several independent empirical procedures (Berlin, et al., 1968). These empirical successes encourage us in the broad view of test (I). Accordingly we take test (I) to be literally applicable to taxa in principle and practically applicable to taxa to the degree that we are able to extend our range of empirical methods for assessing informants' judgments of the truth or falsity of an assertion.

It is theoretically crucial that contrast relations be originally defined upon taxa rather than upon the lexemes that realize them. This essay in effect constitutes an argument in support of that assertion. It is based on the general assumption that significant contrasts in linguistic content are psychologically independent of significant contrasts in linguistic expression but not conversely. To put it in plain, if perhaps imprecise, terms: We speak in order to communicate thought; we do not think in order to provide content for our speech. Perhaps this belief is not totally dissimilar to that which animates a number of contemporary approaches to grammar which, although quite diverse, share a common concern with meaning (e.g. those of Chafe, Fillmore, Lakoff, McCawley, and Ross).

We consider now the frequently used term 'level of contrast' and various derivative expressions. Two kinds of confusion may result from the use of these terms. The first is a confusion of nomenclature (lexemes) and classification (taxa); the second is a conflation between the relations 'having the same level' and 'being in the same contrast set'.

Figure 3 illustrates the way expressions such as 'level of contrast' may lead to self-contradictory or nonsensical statements by confusing taxa with the lexemes that are their names. Parts a and b of Figure 3 diagram the same set of semantic relationships. In speaking of situations such as that pictured in Figure 3, it is sometimes said that "MAN contrasts with ANIMAL at one level and with WOMAN at another level." It is quite unclear, however, whether the expressions MAN, ANIMAL, and WOMAN are supposed to refer to taxa or to the lexemes that realize taxa or, in fact, what the statement means at all. Let us examine the facts of which note is implicitly being taken. These are (1) that the lexeme man is polysemous, being the realization both of t_2 and of t_4; (2) that t_2 is in the same contrast set as t_3 (which is realized by the lexeme animal), while t_4 is in the same contrast set as t_5 (which is realized as woman); (3) that the contrast sets $\{t_2, t_3\}$ and $\{t_4, t_5\}$ are at different levels. Less technically, the lexeme man is polysemous, being the realization of two distinct taxa which are members of distinct contrast sets that also happen to be at different levels. The key fact being noticed is the polysemy of man. (The polysemy of animal is incidental to the example.)

Now compare Figure 4. In this kind of a situation it is also in keeping with common usage to say, "WILD PEPPER contrasts with HOUSEYARD PEPPER at one level and with HOUSEYARD CHILI PEPPER at another level." But again, the intended reference of the capitalized forms is not at all clear. Note particularly that, whereas the key factor in the previous example was the polysemy of the lexeme man, there is no polysemy in this case.

It is true that in the box diagram of Figure 4b the box containing WILD PEPPER is visually "on a level", so to speak, both with the box containing HOUSEYARD PEPPER and the one containing HOUSEYARD CHILI PEPPER. However, this is merely an observation on the visual properties of box diagrams and reflects nothing about the taxonomic structure being pictured. Perhaps this accident of visual imagery has led people into imagining that there is some meaningful sense of taxonomic "level" according to which a taxon (for example t_3--WILD PEPPER--in Figure 4) may be said to have two distinct levels. If so, I have found no indication of what that sense might be. In any case, whatever the expression "X contrast with Y at one level and with Z at another level" means with respect to Figure 3, it cannot mean the same thing as it does with respect to Figure 4; in the former case the reference is to polysemy and in the latter there is no polysemy. In Figure 3 "the same linguistic form [man] designates segregates [the distinct taxa t_2 and t_4] at different levels...", while in Figure 4 "a single unpartitioned segregate [the taxon t_3] contrasts with two or more other segregates [e.g., taxa t_2 and t_4] which are themselves at different levels..." (Frake 1962: 82).

The second confusion arises from the very expression 'level of contrast', because the existence of such a term has suggested to some that any two taxa at the same level are in the same kind of contrast relation. The literature abounds with uses of the expression 'level of

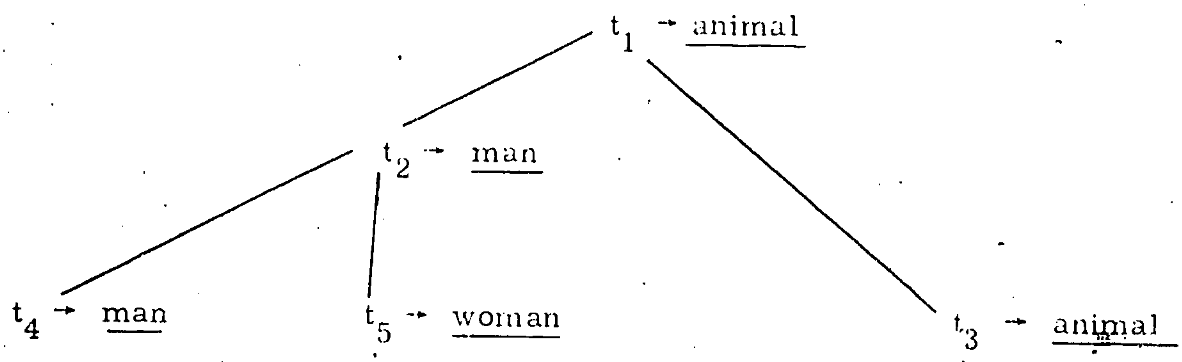


Figure 3a

Adapted from Frake (1961: 117). Arrows connect taxa to the lexemes that are their realizations.

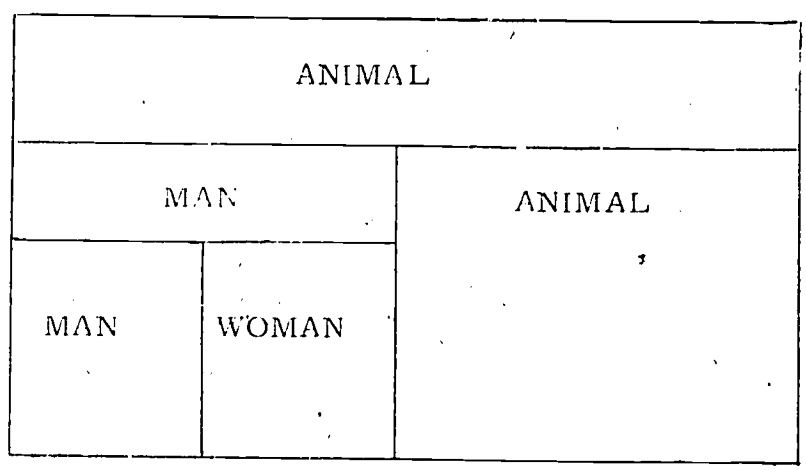


Figure 3b

Box diagram of the same set of relationships as that shown in Figure 3a.

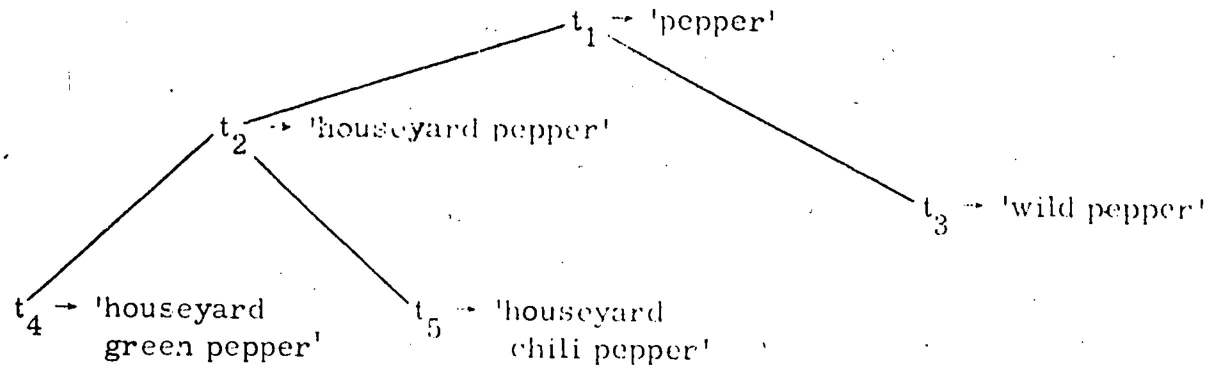


Figure 4a

Adapted from Conklin (1962:131-3)

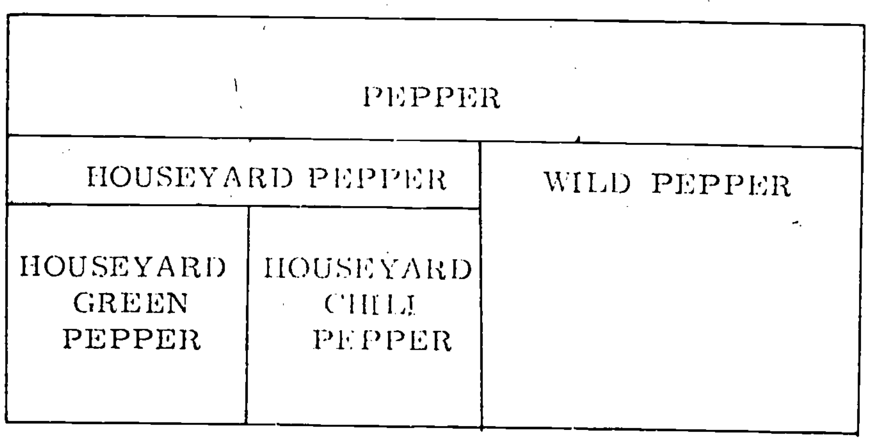


Figure 4b

Box diagram of the same set of relationships as that shown in Figure 4a.

contrast' which implies that all taxa at the same level are in the same contrast set. However, this is obviously not the case,* and it is unlikely that anyone would have been led to this confusion were it not for the currency of the unfortunate expressions 'level of contrast' and 'contrast at the same(different) levels'. Knowing the level at which two taxa occur gives us very little information about the kind of contrast relations that obtain between them. The level of a taxon merely says how many taxa occur between it and the unique beginner in the chain of immediate precedence that connects them. In particular, two distinct taxa which have the same level may or may not be in the same contrast set [Appendix (Remark following Theorem 17)]. (Level one is unique in that all the taxa at this level do constitute a single contrast set.) The expression 'level of contrast', as it has been used in the anthropological literature, is at best ambiguous. In the context of the present formulation it is literally meaningless.

In the hope of avoiding these and similar kinds of terminological confusions with respect to the contrast relations between taxa, we introduce the following definitions of kinds of taxonomic contrast. As we have discussed, two taxa, one of which strictly includes the other (such as tree and oak), do contrast semantically. We name this kind of contrast relation <u>inclusion contrast</u> [Appendix (23)].

A special contrast relation obtains between any two taxa which belong to the same contrast set. We call this relation <u>direct contrast</u>. That is, two taxa are said to contrast directly just in case they are in the same contrast set [Appendix (21)]. Direct contrast is perhaps what users of the expression 'contrast at the same level' have most often had in mind.

Two taxa which are in neither direct contrast nor inclusion contrast are said to be in indirect contrast (or to contrast indirectly) via the two taxa which include them and which are themselves in direct contrast.

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*Cf. Figure 2 and the various numbered statements on page 12.

For example in Figure 5, t_4 and t_6 contrast indirectly via t_2 and t_3 [Appendix (23)]. (Note that in defining inclusion contrast we speak of inclusion, not strict inclusion. Thus, for example, in Figure 5, t_4 and t_3 contrast indirectly via t_2 and t_3.)

We define a special contrast relation which obtains among the terminal taxa. Any two terminal taxa are said to be in terminal contrast. In Figure 5, t_4 and t_9 are in terminal contrast as are also the pairs (t_4, t_5) and (t_4, t_6). Any two terminal taxa are in terminal contrast regardless of whether they are in the same contrast set or at the same level. In Figure 5, t_9 is at level one, while each of the other terminal taxa $t_4 \ldots t_8$ is at level two; nevertheless t_9, is in terminal contrast with each taxon $t_4 \ldots t_8$.

The basis of the intuition 'terminal contrast' is the fact that the terminal taxa collectively furnish the finest possible partition of the unique beginner [Appendix (20)]. That is, the terminal taxa collectively provide the finest available set of mutually exclusive and jointly exhaustive taxonomic categories for classifying an individual. Thus each terminal taxon has a special contrast relation with each other, regardless of its level or contrast set affiliation. To describe individuals in terms of terminal taxa is to slice the taxonomic pie as finely as possible.

The relation defined here as terminal contrast may be related to certain usages of the vague expression 'lowest level of contrast.' It is sometimes said, for example, that certain analyses are only appropriately performed on taxa that "occur at the lowest level of contrast." As we have seen, the 'lowest level of contrast' is meaningless since the expression 'level of contrast' is itself undefined. Specifically, all terminal taxa are not necessarily at the same level. It cannot be emphasized too strongly that the levels at which two taxa occur bear only indirect relevance to the relation(s) of contrast obtaining between them and in no way are sufficient to determine those relations.

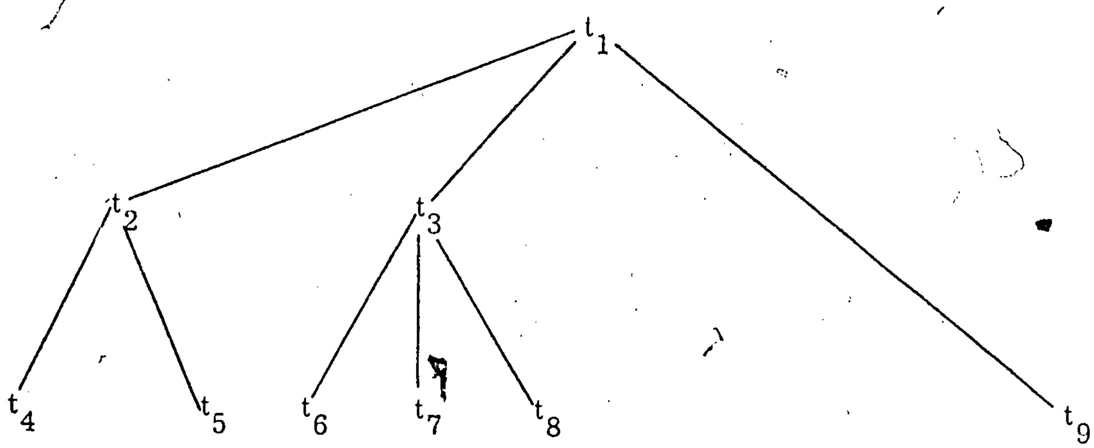

Figure 5

Berlin has recently made a series of remarkable discoveries concerning a subset of taxa which he terms generic (1969 a, b; Berlin, Breedlove, and Laughlin 1969). Generic taxa appear to occupy a privileged status in all natural taxonomies. Substantively, in biotaxonomies they correspond frequently, although not invariably, to Western biological genera. (Although Berlin's researches have so far been restricted to biological taxonomies, I would hazard the speculation that generic categorization may turn out to be a fundamental human thought process.) From a formal point of view, the generic taxa constitute a partition of the unique beginner. They may occur at any non-zero level and may or may not be terminal. As we have already noted, any individual in the taxonomic domain--that is, any member of the unique beginner--is a member of exactly one terminal taxon. Each terminal taxon is in turn linked to the unique beginner by a chain of taxa connected by immediate precedence. Hence, any object that may be characterized by membership in a terminal taxon may, alternatively, be conceptualized by the user of the taxonomic structure in terms of any of the taxa in the appropriate immediate precedence chain. For example, Lassie is a collie, and also a dog, a mammal, a vertebrate, ..., and ultimately an animal. Since the generic taxa partition the domain of relevant individuals , each chain of immediate precedence of this type contains exactly one generic taxon. One of Berlin's important substantive hypotheses deriving from the concept of generic taxon is that one taxon from each such a chain is the most salient and the most frequently employed by actual persons in actual classifying events and that this is the generic taxon. Roughly then, the generic taxa are the ones that partition the domain of individuals in the way that corresponds to the most obvious discontinuities in nature, furnishing a subset of taxonomic categories which are the most obvious, natural, and frequently employed. For example, dog is a generic category in folk English, and a particular dog, say Lassie, is probably more often thought of as a dog than as a collie, or as a mammal, or as a vertebrate. (Certainly she is more often referred to by the lexeme dog.)

The above discussion gives only the barest suggestion of the substantive nature of Berlin's concept of generic taxon, and the reader is referred to the cited works for full explanation of generic taxa, their relation to taxonomic nomenclature, and their role in the synchronic use and diachronic development of taxonomies. My sole purpose here is to indicate the existence of empirical motivation for the formal recognition of an additional type of contrast relation, based on the psychological importance of the generic partition. Generic contrast is defined simply as that special relation of contrast which holds between any two generic taxa [Appendix (29, 30)]. Thus, just as terminal contrast is based upon the finest partition of the domain of individuals available to the user, generic contrast is based upon the most natural and psychologically salient partition.

We have defined five special types of contrast relations. Three of these--inclusion, direct, and indirect contrast--form a natural group in that any pair of taxa in a taxonomic structure are related to each other in exactly one of these ways [Appendix (24)]. The remaining two types of contrast relations--generic and terminal contrast--are also a natural set. Both are based on membership in special sets of taxa that constitute partitions of the domain of individuals, i.e., the terminal and generic partitions. Since the generic taxa may or may not be terminal contrast (and conversely), two taxa in generic contrast may or may not also be in terminal contrast (and conversely) [Appendix (31)]. Two taxa that are in either generic or terminal contrast (or both) are never in inclusion contrast and therefore they must contrast either directly or indirectly [Appendix (24, 26, 31)].

Section 3

As mentioned in the introduction, the intuitive notion of taxonomy involves a taxonomic structure, a set of names (lexemes), and a mapping that carries the former onto the latter. The image of a taxon under this mapping is its lexical realization. However, we do not feel that just any mapping of taxa onto lexemes will model in a revealing way the class of empirical objects we wish to call taxonomies. The problem is that just any mapping does not provide enough structure to capture our intuition of what empirical taxonomies are like. Figure 6 provides some hypothetical examples of mappings of taxa onto sets of lexemes that most people will, I am sure, not wish to call taxonomies.

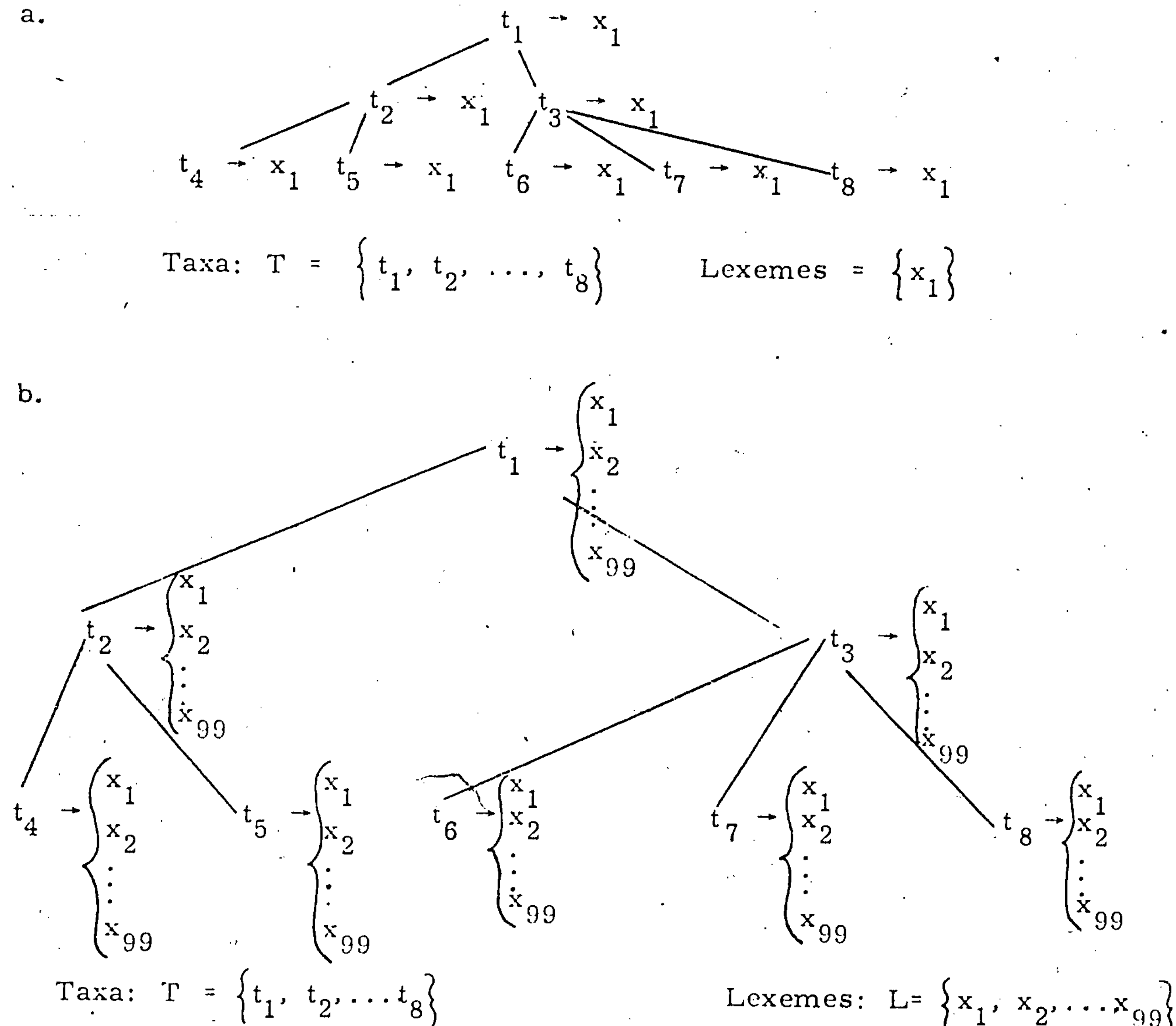

Figure 6

In Figure 6a there is a single name (the lexeme x_1) which is the image of each taxon in the taxonomic structure; hence, none of the structural relations among the taxa are preserved under the mapping A parallel case would occur in English if it were to contain a single word, say fern, to refer to every taxon in the plant world. Fern would thus have the various significations plant , tree , grass , oak , lichen , American Beauty rose , and so on, for each plant category in our culture. It would be preferable if our formal definition of taxonomy ruled out such absurd cases.

Figure 6b pictures a different but equally unsatisfactory situation. Here ninety-nine lexemes are employed to denote eight conceptual categories, and each of the names can refer to any of the categories. Each of the ninety-nine lexemes is thus eight ways polysemous and each has the same denotation as the other. Again the underlying taxonomic structure is totally obscured.

Absurdities such as these can be concocted at will so long as we introduce no constraints on the mapping of taxa onto lexemes. Clearly some constraints must be introduced if the formalism is to be narrow enough accurately to reflect the amount and kind of structure we feel intuitively to exist in real taxonomies. The problem here is to constrain the formal definition of the lexical realization mapping in just the right way--that is in the way that results in a formal definition of 'taxonomy' which fits just those empirically observed structures that we wish intuitively to call taxonomies. In particular we would like our formal structure to be just general enough to admit as examples the natural taxonomies that have been described in detail, such as those referred to in the introduction.

The strongest and mathematically "simplest" constraint requires that the mapping be "one-to-one onto". That is, each taxon is realized by a unique and distinct lexeme. (The "onto" provision simply means that each lexeme realizes some taxon in the relevant taxonomic structure.)

All possibilities of synonymy (more than one lexeme per taxon) and multiple meaning (more than one taxon per lexeme) are excluded. It is very easy to find empirical counterexamples to the no-synonymy condition. For example, in one dialect of Southern Louisiana the forms <u>bass</u>, <u>black bass</u>, <u>trout</u>, and <u>green trout</u> may all be applied to the same taxon and a similar relation holds within other such sets of lexemes as {<u>sac-à-lait</u>, <u>white perch</u>, <u>crappie</u>} and {<u>perch</u>, <u>bream</u>, <u>sunfish</u>}. We cannot accept the no-synonymy constraint.

We may relax the no-synonymy condition by allowing the mapping to be one-many. This model allows for synonymy (more than one lexeme applying to a given taxon) but still prohibits all multiple meaning. That is, it prohibits a given lexeme from being the realization more than one taxon. However, one of the principle achievements of empirical work on taxonomies in the last decade has been the discovery that a given lexeme may in fact realize more than one taxon. As Frake points out (1961: 119), the English lexeme <u>man</u> is used as a name for a variety of taxa which might be glossed 'human', 'male human', 'adult male human', and 'virile adult male human'. Characteristically this sort of multiple meaning involves a proper subset of T whose members can be arranged in a sequence of immediate precedence. In other words, multiple meaning in natural taxonomies appears to be restricted to a special subtype of polysemy with the following property: If a lexeme is polysemous in a taxonomy, its various senses always correspond to taxa which can be arranged in a sequence of immediate precedence. This is the sense of definition (32), including Axiom (33) of the Appendix.

We therefore define taxonomy as follows: A taxonomy consists of (i) a taxonomic structure, (ii) a finite set of lexemes, and (iii) a mapping of the former onto the latter which allows synonymy freely and which allows polysemy only over sets of taxa that can be linked in chains of immediate precedence [Appendix (32, 33)].

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APPENDIX I

(1) Def. A binary relational structure $\tau = \langle T, \supset \rangle$, where T is a finite non-null set of non-null sets and $\supset$ is the relation of strict inclusion restricted to the members of T, is a taxonomic structure iff[1] the following two axioms are satisfied.

(2) Axiom There is a unique $t_1 \in T$, called the unique beginner, such that, for any $t_i \in T$ $(t_i \neq t_1)$, $t_1 \supset t_i$.

(3) Axiom For any $t_i \in T$, the set $c(t_i) = \{t_j \mid t_j \in T, t_i \supset t_j$ and there is no $t_k \in T$ such that $t_i \supset t_k$ and $t_k \supset t_j\}$ is either null or is a partition of t_i.

(4) Def Immediate Precedence $= \{ \langle t_i, t_j \rangle \mid t_i, t_j \in T, t_i \supset t_j$ and there is no $t_k \in T$ such that $t_i \supset t_k$ and $t_k \supset t_j \}$.

REMARK: Thus in axiom (3), $c(t_i) = \{ t_j \mid t_i \rightarrow t_j \}$.

If $\langle t_i, t_j \rangle$ is a member of Immediate Precedence, we write equivalently "$t_i \rightarrow t_j$", "t_i immediately preceeds t_j", "t_i is the immediate precedessor of t_j", "t_j immediately succeeds t_i", and so on. If $\langle t_i, t_j \rangle \notin$ Immediate Precedence we write $t_i \nrightarrow t_j$.

The following properties of Immediate Precedence (5.i-iv) follow directly from Definition (5), the well-known properties of proper inclusion of sets, and the fact that T is finite (Def. 1).

(5) For any distinct $t_i, t_j, t_r \in T$

(i) $t_i \nrightarrow t_i$,

(ii) If $t_i \rightarrow t_j$, then $t_j \nrightarrow t_i$,

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[1] Read "if and only if."

(iii) If $t_i \rightarrow t_k$ and $t_k \rightarrow t_j$ then $t_i \nrightarrow t_j$,

(iv) If $t_i \supset t_j$ and $t_i \nrightarrow t_j$ then there is a $t_k \in T$ such that $t_i \rightarrow t_k$ and $t_k \supset t_j$.

(6)Theorem For any t_i, $t_j \in T$, if $t_i \supset t_j$ and $t_i \nrightarrow t_j$ then there is a <u>unique</u> $t_k \in T$ such that $t_i \rightarrow t_k$ and $t_k \supset t_j$.

Proof:

The existence of t_k is guaranteed by (5. iv). Hence, by Axiom (3), $c(t_i)$ is non-null. The uniqueness of t_k now follows from Axiom (3). which ensures that (a) $t_k \in c\,(t_i)$ and (b) $c(t_i)$ is a partition of t_i'. The proof is complete.

Note on notation: For convenience we abbreviate

'$t_a \rightarrow t_b$, $t_b \rightarrow t_c$, $t_c \rightarrow t_d$, ... ' as

'$t_a \rightarrow t_b \rightarrow t_c \rightarrow t_d$, ...' . One should bear in mind that '$t_a \rightarrow t_b \rightarrow t_c$' does <u>not</u> mean $t_a \rightarrow t_c$; on the contrary, it implies $t_a \nrightarrow t_c$ (see 5.iii). (a, b) = (b, a); $\langle a, b\rangle \neq \langle b, a\rangle$; 'ϕ' denotes the null set.

(7)Theorem For any t_i, $t_j \in T$, if $t_i \supset t_j$ and $t_i \nrightarrow t_j$ there exists a unique finite sequence of distinct elements of T

$t_i \rightarrow t_k \rightarrow \ldots \rightarrow t_j$.

Proof:

From Theorem (6), there is a unique $t_k \in T$, $t_i \rightarrow t_k$ and $t_k \supset t_j$. By hypothesis $t_i \nrightarrow t_j$; hence, $t_k \neq t_j$. Either $t_k \rightarrow t_j$ or $t_k \nrightarrow t_j$. If $t_k \rightarrow t_j$, the desired sequence is established, namely $t_i \rightarrow t_k \rightarrow t_j$. If $t_k \nrightarrow t_j$, by applying Theorem (7) again, we establish a unique $t_m \in T$ such that $t_k \rightarrow t_m$ and $t_m \supset t_j$. If $t_m \rightarrow t_j$, we have the desired sequence, namely $t_i \rightarrow t_k \rightarrow t_m \rightarrow t_j$. If $t_m \nrightarrow t_j$, the next term of the sequence is uniquely established by applying theorem (6) to t_m and t_j. This procedure is continued until the term $t_n \in T$ $(t_n \rightarrow t_j)$ is found and the sequence is complete.

Since (1) no element t_r of T can occur more than once in the sequence ($t_r \rightarrow \ldots \rightarrow t_r$ implies $t_r \supset t_r$, which is absurd) and (2) the sequence is finite (because T is itself finite; Definition 1), we are assured of finding the required $t_n \in T$, $t_i \supset t_n$, $t_n \rightarrow t_j$. This completes the proof.

(8)Def. A subset s_i of T is a <u>contrast set</u> iff there is a $t_i \in T$ such that $c(t_i) \neq \Phi$ and $s_i = c(t_i)$.

(9)Theorem Every contrast set is a proper subset of T.

Proof: For any $t_i \in T$, $t_i \notin c(t_i)$ (Axiom 3). This completes the proof.

(10)Theorem Every contrast set has a least two members.

Proof: Assume the contrary: there are t_i, $t_j \in T$ such that t_j is the unique member of $c(t_i)$. Since $c(t_i)$ is a partition of t_i (Axiom 3), $t_j = t_i$. However, also by Axiom (3), $t_i \rightarrow t_j$. Hence $t_i \neq t_j$ (5.i). Thus a contradiction is established and the proof is complete.

(11) Def. For any $t_i \in T$, t_i is terminal iff $c(t_i) = \Phi$.

(12)Theorem For any $t_j \in T$ ($t_j \neq t_1$) there is a unique $t_i \in T$, $t_i \rightarrow t_j$.

Proof: By Axiom (2) $t_1 \supset t_j$. Thus, this theorem follows directly from the special case of Theorem (8) which specifies the unique sequence $t_1 \rightarrow \ldots \rightarrow t_j$.

(13) Def. For any $t_j \in T$,

(i) <u>t_j has level zero</u> [$L(t_j) = 0$] iff $t_j = t_1$;

(ii) <u>t_j has level $n+1$</u> [$L(t_j) = n+1$] iff $L(t_i) = n$ and $t_i \rightarrow t_j$ ($t_i \in T$, n is a non-negative integer).

(14)Remark The preceding definition is justified by the following fact.

For any $t_j \in T$, there is a unique non-negative integer n such that $L(t_j) = n$.

Proof: If $t_j = t_1$, $L(t_j) = 0$ according to (13.i). If $t_j \neq t_1$, the uniqueness of $n = L(t_j)$ follows from Theorem (12) and Definition (13.ii).

(15)Theorem For distinct $t_i, t_j \in T$; $t_i \supset t_j$ iff t_i occurs in the sequence $t_1 \to \ldots \to t_j$.

Proof: Suppose $t_i \supset t_j$. Then by Theorem (7) there is a unique sequence $t_i \to \ldots \to t_j$. If $t_i = t_1$, then clearly t_i occurs in the sequence $t_1 \to \ldots \to t_j$. If $t_i \neq t_1$ and if t_i were not in the sequence $t_1 \to \ldots \to t_j$, then the sequence $t_1 \to \ldots \to t_j$ would not be unique, since there would be distinct sequences $t_1 \to \ldots \to t_i \to \ldots \to t_j$ and $t_1 \to \ldots \to t_j$. Hence if $t_i \supset t_j$, t_i must occur in the sequence $t_1 \to \ldots \to t_j$, and the first half of the proof is complete. If t_i is in the sequence $t_1 \to \ldots \to t_j$, and $t_i \neq t_j$, then either $t_i \to t_j$ or there is a unique $t_k \in T$, $t_i \supset t_k$ and $t_k \to t_j$ (Theorem 6). Since $t_k \to t_j$ implies $t_k \supset t_j$ (Definition 4) and since '$\supset$' is transitive, in either case $t_k \supset t_j$. The proof is now complete.

(16)Theorem For distinct $t_i, t_j \in T$, either $t_i \supset t_j$ or $t_j \supset t_i$ or t_i and t_j are disjoint.

Proof:
If either t_i or t_j is the unique beginner then it properly includes the other (Axiom 2). Otherwise there are unique sequences $t_1 \to \ldots \to t_i$ and $t_1 \to \ldots \to t_j$. (a) $t_i \supset t_j$ iff t_i occurs in the sequence $t_1 \to \ldots \to t_j$ (Theorem 15). Similarly (b) $t_j \supset t_i$ iff t_j occurs in the sequence $t_1 \to \ldots \to t_i$. It only remains to be shown that if neither condition (a) nor condition (b) holds, then t_i and t_j are disjoint. Consider $t_p, t_q \in T$, $t_p \supseteq t_i$, $t_q \supseteq t_j$, and

$t_1 \to t_p$, $t_1 \to t_q$. If $t_p \neq t_q$, then t_p and t_q are disjoint, since t_p , $t_q \in c(t_1)$ (Axiom 3). If t_p and t_q are disjoint, then t_i and t_j are disjoint. If $t_p = t_q$, then consider t_r , $t_s \in T$, $t_p = t_q \to t_r$, $t_p = t_q \to t_s$, $t_r \supset t_i$, $t_s \supset t_j$. Again if $t_r \neq t_s$, then t_r and t_s are disjoint because t_r , $t_s \in c(t_p) = c(t_q)$, and consequently t_i and t_j are disjoint. This procedure is continued until t_m , $t_n \in T(t_m \supset t_i$, $t_n \supset t_j$, $t_m \wedge t_n = \Phi)$ are discovered. Such t_m , t_n must eventually be discovered if, as we have assumed, t_i does not occur in $t_1 \to \ldots \to t_j$ and t_j does not occur in $t_1 \to \ldots \to t_i$. Since $t_m \wedge t_n = \Phi$ and $t_m \supset t_i$ and $t_n \supset t_j$, then $t_i \wedge t_j = \Phi$. The proof is thus complete.

(17)Theorem For any distinct t_i , $t_j \in T$, if there is a $t_k \in T$ such that t_i , $t_j \in c(t_k)$, then $L(t_i) = L(t_j)$.

Proof:

By Theorem (14), $L(t_k)$ is unique; let $L(t_k) = m$. Then by (13.ii) $L(t_i) = L(t_j) = m + 1$.

REMARK:

The converse of Theorem (17) does not hold in general. That is, the following statement is not true: For any t_i , $t_j \in T$, if $L(t_i) = L(t_j)$, then t_i and t_j are in the same contrast set.

We demonstrate this by counterexample. Consider $T = \{t_1 , t_2 , t_3 , t_4 , t_5 , t_6 , t_7\}$ where $t_1 \to t_2 \to t_4$, $t_1 \to t_2 \to t_5$, $t_1 \to t_3 \to t_6$, $t_1 \to t_3 \to t_7$. Clearly, the axioms, (2) and (3), are satisfied. $L(_4) = L(t_6) = 2$. However, there is no $t_i \in T$ such that $t_i \to t_4$ and $t_i \to t_6$. That is, although t_4 and t_6 are at the same level, they are not in the same contrast set.

(18)Def. A taxonomic structure $\tau = \langle T , \supset \rangle$ has <u>depth</u> n iff there is a $t_i \in T$ such that $L(t_i) = n$ and there is no $t_j \in T$ such that

$L(t_j) > n$.

(19) Theorem If a taxonomic structure $\tau = \langle T, \supset \rangle$ is of depth n, then for each integer m $(0 \leq m \leq n)$ there is at least one $t_i \in T$ such that $L(t_i) = m$.

Proof:
By definition (18) there is some $t_j \in T$ such that $L(t_j) = n$. Consider the sequence $t_1 \to \ldots \to t_j$. By applying the recursive definition of level (13) to succeeding terms in this sequence we establish that the sequence has $n + 1$ terms and the i^{th} term has level $i - 1$. The proof is complete.

(20) Theorem The subset $S_a = \{ t_i \mid t_i \in T$ and t_i is terminal$\}$ of T is a partition of t_1 .

Proof:
If $t_i, t_j \in T$ are both terminal, then by definition (11) neither $t_i \supset t_j$ nor $t_j \supset t_i$. Hence, by Theorem (16), any distinct $t_i, t_j \in T$ which are terminal are disjoint. It remains to be shown that the union of all $t_i \in S_a$ exhausts t_1 . For an arbitrary individual x, if $x \in t_1$ then there must be a $t_i \in T$ such that $L(t_i) = 1$ and $x \in t_i$ (Axiom 3). Now, either $c(t_i)$ is null or it is a partition of t_i . (Axiom 3). Hence either t_i is terminal and $x \in t_i$ or there is exactly one $t_j \in T$ such that $t_j \in c(t_i)$ and $x \in t_j$. Again t_j may either be terminal, in which case there is a terminal member of T that contains the arbitrary individual x, or x is a member of exactly one member t_k of $c(t_j)$. Continuing in this way, we eventually find a terminal t_n such that $x \in t_n$ (since T is finite). We have thus established that for an arbitrary individual $x \in t_1$, there is a terminal t_n such that $x \in t_n$. That is, the $t_i \in S_a$ exhaust t_1 . The proof is thus complete.

(21) Def. Direct Contrast = $\{ (t_i , t_j) \mid t_i , t_j \in T , t_i \neq t_j , t_i$ and t_j are in the same contrast set$\}$.

(22) Def. Indirect Contrast = $\{ (t_i , t_j) \mid t_i , t_j \in T$ and there are $t_m , t_n \in T$ such that $t_m \supseteq t_i$ and $t_n \supseteq t_j$ and $(t_m , t_n) \in$ Direct Contrast and either $t_m \supset t_i$ or $t_n \supset t_j \}$.

If $(t_i , t_j) \in$ Direct Contrast, we say equivalently "t_i and t_j contrast directly", "t_i and t_j are in direct contrast", "t_i and t_j are in the same contrast set." If $(t_i , t_j) \in$ Indirect Contrast, we say "t_i and t_j contrast indirectly via t_m and t_n respectively", "t_j and t_i contrast indirectly via t_m and t_n respectively", "t_i and t_j contrast indirectly."

(23) Def. Inclusion Contrast = $\{ (t_i , t_j) \mid t_i , t_j \in T$ and either $t_i \supset t_j$ or $t_j \supset t_i \}$.

If $(t_i , t_j) \in$ Inclusion Contrast, we say equivalently "t_i and t_j are in the relation of inclusion contrast," "t_i and t_j contrast inclusively."

(24) Theorem For any distinct $t_i , t_j \in T$, one and only one of the following

(i) t_i and t_j contrast inclusively,

(ii) t_i and t_j contrast directly,

(iii) t_i and t_j contrast indirectly.

Proof:

We show first that conditions (i), (ii), (iii) are pairwise contradictory, that is, that the sets Direct Contrast, Indirect Contrast, and Inclusion Contrast are mutually exclusive. Conditions (ii) and (iii) are contradictory because of the provision in Definition (22) that "either $t_m \supset t_i$ or $t_n \supset t_j$." Conditions (i) and (ii) are contradictory because, if t_i and t_j are in the same contrast set, then neither $t_i \supset t_j$ nor $t_j \supset t_i$, and conversely. (Axiom 3, Definition 9). Conditions (i) and (iii) are contradictory because if t_m and t_n contrast directly then $t_m \wedge t_n = \Phi$. Then since $t_m \supset t_i$ and $t_n \supset t_j$, $t_i \wedge t_j = \Phi$. Consequently neither $t_i \supset t_j$ nor $t_j \supset t_i$. If $t_i \supset t_j$ or $t_j \supset t_i$, then t_i and t_j are obviously not in the same contrast set. It remains only to be shown that either (i), (ii), or (iii) holds for every pair of distinct $t_i , t_j \in T$. Consider the sequences $t_1 \rightarrow \ldots \rightarrow t_r \rightarrow t_i$ and $t_1 \rightarrow \ldots \rightarrow t_s \rightarrow t_j$. If t_i occurs in the latter or t_j in the former, then (i) holds for t_i , t_j. Iff $t_r = t_s$, (ii) holds for t_i , t_j. If neither (i) nor (ii) hold, we read down the two sequences in the way we did in the proof of Theorem (16), eventually discovering t_m and t_n such that $t_m \supset t_i$, $t_n \supset t_j$, and t_m , t_n are in direct contrast. Thus, at least one of the three conditions (i), (ii), (iii) must hold for any $t_i , t_j \in T$. The proof is complete.

(25)Def. Terminal Contrast = $\{(t_i , t_j) \mid t_i , t_j \in T , t_i \neq t_j , t_i$ and t_j are each terminal$\}$. If $(t_i , t_j) \in$ Terminal Contrast we say equivalently "t_i and t_j are in terminal contrast", "t_i and t_j contrast terminally".

26)Theorem For any $t_i , t_j \in T$, if t_i and are in terminal contrast then t_i and t_j are not in inclusion contrast.

Proof:

By Definitions (11) and (25), there is no $t_k \in T$ such that $t_i \supset t_k$, and similarly there is no $t_k \in T$ such that $t_j \supset t_k$. Hence neither $t_i \supset t_j$ nor $t_j \supset t_i$. The proof is complete.

REMARK:

For any $t_i , t_j \in T$, if t_i and t_j are in terminal contrast then

(i) t_i and t_j may or may not contrast directly,

(ii) t_i and t_j may or may not contrast indirectly.

We demonstrate this remark by examples. Assume

$T = \{ t_1 , t_2 , t_3 , t_4 , t_5 \}$ where $t_1 \to t_2 \to t_3$, $t_1 \to t_2 \to t_4$, $t_1 \to t_5$

Axioms (2, 3) are satisfied.

(i) t_3 and t_4 contrast terminally and also directly; t_4 and t_5 contrast terminally but do not contrast directly.

(ii) t_4 and t_5 contrast terminally and also indirectly; t_3 and t_4 contrast terminally but do not contrast indirectly. The demonstration is complete.

(28) Theorem For any $t_j \in T$ $(t_j \neq t_1)$ there is exactly one contrast set $c(t_i)$ $(t_i \in T)$ such that $t_j \in c(t_i)$.

Proof:

This theorem follows directly from Axiom (3) and Theorem (12).

REMARK:

In an arbitrary taxonomic structure $\tau = \langle T , \supset \rangle$, it is not true in general that, for any terminal t_i and t_j $(t_i , t_j \in T)$, $L(t_i) = L(t_j)$.

The previous remark is established by example, as follows:

Consider $T = \{ t_1 , t_2 , t_3 , t_4 , t_5 \}$ where $t_1 \to t_2 \to t_3$, $t_1 \to t_2 \to t_4$, $t_1 \to t_5$. Axioms (2) and (3) are satisfied.

t_4 and t_5 are terminal, but $L(t_4) = 2$ while $L(t_5) = 1$.

(29) Def. There is a unique, non-null subset G of T which partitions t_1 and whose members $g_i \neq t_1$ are called generic. Remark: In each empirical case, the generic taxa must be isolated by empirical operations, subject only to the above conditions.

(30) Def. Generic Contrast = $\{(t_i, t_j) \mid t_i, t_j \in T, t_i \neq t_j, t_i \text{ and } t_j \text{ are each generic}\}$.

(31) Theorem For distinct $t_i , t_j \in T$, if $(t_i , t_j) \in$ generic contrast, then t_i and t_j do not contrast inclusively.

Proof: the theorem follows directly from Definitions (23) and (30).

Remark: For any $t_i , t_j \in T$, if t_i and t_j are generic contrast, then

(i) t_i and t_j may or may not contrast directly

(ii) t_i and t_j may or may not contrast indirectly

(iii) t_i and t_j may or may not contrast terminally.

The demonstration of the REMARK that follows Theorem (26) serves also as a demonstration (a) for parts (i) and (ii) if we assume that the terminal taxa in that example (t_3, t_4, and t_5) are also the generic taxa and (b) for part (iii) if we assume alternatively that t_2 and t_5 are the generic taxa.

(32) Def. A taxonomy is a trinary relational structure $\mathcal{J} = \langle \Upsilon, L, m \rangle$ where

(i) $\Upsilon = \langle T, \supset \rangle$ is a taxonomic structure

(ii) $L = \{x_1, x_2, \ldots, x_r\}$ is a finite set of lexemes and

(iii) m is a mapping of T onto L which satisfies the following axiom.

(33) Axiom If distinct t_i, $t_j \in T$ each have the image x_i under m then

(i) either $t_i \supset t_j$ or $t_j \supset t_i$, and

(ii) for each $t_k \in T$, if $t_i \supseteq t_k \supseteq t_j$ or $t_j \supseteq t_k \supseteq t_i$, then t_k also has the image x_i under m.

ADDENDUM

After the present essay was completed, I came upon Gregg's latest formulation of the problem. Gregg (1967) defines finite n-rank Linnaean structures.* These mathematical objects are similar in many ways to taxonomic structures as defined above although they differ in some important respects. The following comparative discussion is both brief and informal.

Gregg's n-rank Linnaean structures are roughly comparable to taxonomic structures of depth n. Both involve a finite set of taxa arranged in a hierarchy of immediate precedence with a unique beginner. (In terms of graph theory, either may be represented by a digraph in the form of a rooted tree. Taxonomic structures add the restriction, absent from Gregg's Linnaean structures, that no vertex has a positive degree of one. The latter condition reflects the fact that taxonomic structures do not allow monotypic taxa; i.e., if a taxon has subtaxa, it has at least two of them.) Gregg's structures may be <u>open</u> or <u>not open</u>. There is nothing the theory of taxonomic structure corresponding to Gregg's open Linnaean structures. Gregg's theory is thus the more general one. An open Linnaean structure is one in which the unique beginner contains at least one member that is contained in no terminal taxon. Thus in an open structure there is at least one taxon that <u>strictly</u> includes the union of the members of the contrast set it dominates. In symbols, there is in each open Linnaean structure at least one taxon t_i such that

(1) $$t_i \supset \bigcup_{t_j \varepsilon c(t_i)} t_j .$$

This condition violates Axiom II, which, it will be recalled, requires that $c(t_i)$ be a partition of t_i for all t_i in T. It is not surprising that taxonomic structures are closed systems in Gregg's sense considering that taxonomic

*I am indebted to Brent Berlin for bringing this work to my attention.

structures prohibit monotypic taxa and that the major motivation for Gregg's introduction of the notion of openness is to deal with the "problem of monotypy" (Gregg 1967: 204; italics in original). I will return briefly to Gregg's problem of monotypy below.

The remaining major differences between Linnaean structures as defined and classified by Gregg and taxonomic structures involve the notion of absolute taxonomic category. The notion of an absolute scale of generality of taxa--constituted by a series of categories such as species, genus, family, order, class, etc. and which assigns each taxon to one such category--plays a crucial role in Gregg's formulation but is not present as such in the present development. For Gregg each category is a set of disjoint taxa. An n-rank Linnaean structure contains n categories; the unique beginner is the sole member of category n, and each category C_i ($1 \leq i \leq n$) contains at least one taxon. Each taxon belongs to exactly one category, but the category of a taxon is not necessarily exactly one less than that of its immediate predecessor; nor are all terminal taxa necessarily in category one. If a Linnaean structure contains one or more taxa whose category is not exactly one less than that of its immediate predecessor, Gregg calls such a structure irregular. If a Linnaean structure contains one or more terminal taxa that are not in category one, Gregg terms it a truncated structure. Figure 1a depicts a truncated but regular, closed, 3-rank Linnaean structure (Gregg prefers "not irregular", and also "not truncated", "not open", 1967:195 ff). Figure 1b depicts a non-truncated, irregular, closed, 3 rank Linnaean structure. A glance at the figures will reveal that these structures are identical from the point of view of taxonomic structure. A Linnaean structure must be either truncated or irregular (or both) if its terminal taxa are not all at the same level (as level is defined in taxonomic structure).

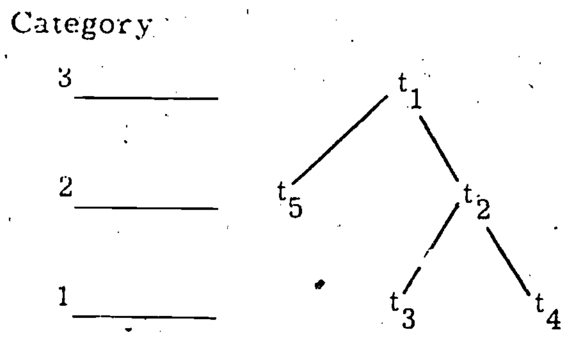

Figure 1a

Abstract 3-rank, closed, truncated, regular Linnaean Structure

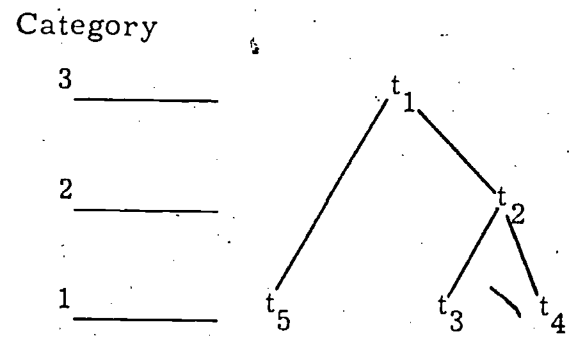

Figure 1b

Abstract 3-rank, closed, non-truncated, irregular Linnaean Structure

For completeness, Figures 2a and 2b show 3-rank Linnaean Structures that are (a) truncated and irregular and (b) non-truncated and regular, respectively.

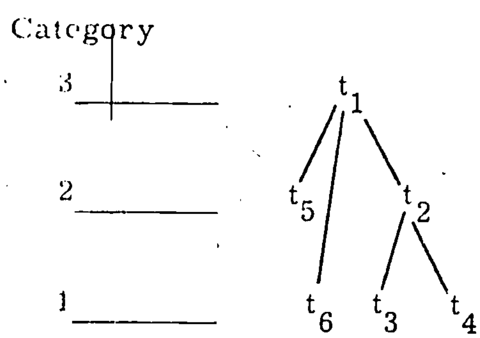

Figure 2a

Abstract 3-rank closed, truncated, irregular Linnaean Structure

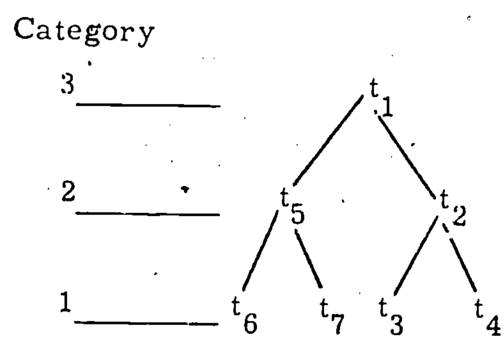

Figure 2b

Abstract 3-rank, closed, non-truncated, regular Linnaean Structure

Figure 2b (non-truncated, regular) corresponds of course to a taxonomic structure all of whose terminal taxa are at the same level. Recalling that it is possible but by no means necessary that all the terminal taxa in a taxonomic structure have the same level, we arrive the following general comparison of n-rank Linnaean structures and taxonomic structures.

1. Both a Linnaean structure and a taxonomic structure consist of a finite set of taxa, hierarchically ordered by the relation of immediate precedence. (For a useful formal definition of hierarchy, see Gregg 1967:194 f).

2. Open Linnaean structures violate Axiom II of the theory of taxonomic structure. Taxonomic structures are thus, in Gregg's sense, closed.

3. Gregg's notions of truncation and regularity, being based on a series of absolute, ranked categories, are not directly applicable to taxonomic structures. Any closed, n-rank Linnaean structure thus determines a unique taxonomic structure (of depth n).

4. The converse of the last remark does not hold; that is, a given taxonomic structure does not determine a unique Linnaean structure. For example, Figures 1a and 1b diagram two distinct Linnaean structures but a single taxonomic structure.

Gregg's explicit motivation for the introduction of the notion of openess is to deal with what he calls the problem of monotypy (1967:201 ff, especially 204). Gregg offers as an example of monotypy, the fact that in Simpson's classification of mammals (1945) the subclass protheria contains but a single order, montremata. Gregg proves that any structure containing monotypic taxa is open. (The converse does not hold. It is not the case that any open structure necessarily contains monotypic taxa. However, there would seem to be only the flimsiest of empirical motivation, if any, in support of the formal notion of an open Linnaean structure without monotypy.) In the case of the protherians and the monotremes, Gregg's formulation commits the biologist to the empirical claim that there are some protherians that are not monotremes. Similarly Gregg's formulation forces us to maintain that there are some members of the family Ornithorhychidae that are not members of the genus Ornithorhychus (1967:202f). Gregg does not inform us whether or not the organisms in question are platypuses. Thus although Gregg accurately characterizes previous serious attempts to resolve the problem of monotypy--such as those of Beckner (1959), Sklar (1964), and van Valen (1964)--as examples of "technical artifice" (Gregg 1967:205), it appears that his own attempt is hardly immune from that charge. Indeed, after presenting the platypus example Gregg says (1967:203), "At this point, it is only fair to mention that the version of monotypy demanded by our model may not be generally acceptable to taxonomists; but, for the moment we shall postpone discussion of the possible difficulties that are raised." I do not know if Gregg has subsequently attempted to discuss the difficulties associated with this entirely counter-intuitive notion of monotypy. A priori they would appear insuperable.

What Gregg treats as the problem of monotypy is treated in the theory of taxonomic structure and taxonomic nomenclature as a problem of polysemy. The present theory rejects the notion of absolute category (except for the gen[illegible] partition) and recognizes the fact that under certain conditions, taxonomists, be they Professors of biology or pursuers of bisons, are liable to use a single name for distinct taxa that can be arranged in a chain of immediate precedence. Whether or not the present formulation turns out to be useful to the biological systematist (for whom it was not originally intended) is a matter for the biological systematist to judge. No claim is made [illegible] "[illegible]." In any case it does seem fairly clear that Gregg's (1967) formulation cannot sustain any such claim.

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